647510 **BSc5MatC501x** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination

March/April-2023 (NEW COURSE)

Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1(A) Answer any two out of three. **[10]**

(1) State and prove the Housedorff's principle for metric space.

(2) X is metric space. Show that intersection of finite numbers of open subsets of X is open. Explain with example that this statement is not true for infinite numbers of open subsets of X.

(3) Explain cantor set and Show that $\frac{1}{10} \in C$.

Q-1(B) Answer any one out of two. **[04]**

(1) (i) is $(A \cup B)^0 = A^0 \cup B^0$? Explain with your answer.

(ii) Explain with example that $\overline{A \cap B} \neq \overline{A} \cap \overline{B}$.

(2) (i) R is discrete metric space. Find N(2,0.5) in R.

(ii) Check whether set $E=(-\infty,10)$ is open and closed.

Q-2(A) Answer any two out of three. **[10]**

(1) Prove that any bounded function f defined on [a,b] is R-integrable iff For $\forall\ \varepsilon>0$ there exist partition P of [a,b] such that $U(P,f) - L(P,f) < \varepsilon$, Where $\|P\| < \delta$.

(2) If bounded functions f is R-integrable in [a,b] then show that |f| is also R-integrable and $|\int_a^b f dx| \le \int_a^b | f | dx$.

(3) Function f is bounded in [a,b]. P and P^* be two partitions of [a,b] Such that $P \subset P^*$ Then show that $U(P^*,f) \le U(P,f)$.

Q-2(B) Answer any one out of two. **[04]**

(1) Find $\int_1^2 (3x+1)\, dx$ using definition of R-integration.

(2) if bounded function f on [a,b] is continuous then show that f is R-integrable in [a,b].

Q-3(A) Answer any two out of three. **[10]**

(1) Using darbour's theorem find value of $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \ldots\ldots\ldots\ldots + \left(\frac{n}{n}\right)^2\right]$

(2) State and prove general form of first mean value theorem for R-integration.

(3) Show that set of all 4^{th} roots of 1forms a group under multiplication and find order of elements of this group.

Q-3(B) Answer any one out of two. **[04]**

(1) Show that $\frac{\pi^3}{51} \le \int_0^\pi \frac{x^2}{10+7\cos(x)}\, dx \le \frac{\pi^3}{9}$

(2) G=R-{-1}and operation * is define on G as: for $\forall\ a,b \in G$ a*b=a+b+ab. Check whether (G,*) is group. If G is group then find 2^{-1} in G.

Q-4(A) Answer any two out of three. **[10]**

(1) Prove that every permutation of finite set is a product of disjoint cycles.

(2) Define: Symmetric group, Alternative group. Find number of elements of Symmetric group S_n. Also write all even permutations of S_3.

(3) State and prove Lagrange's theorem.

Q- 4(B) Answer any one out of two. **[04]**

(1) $G=\{(a,b)/a,b\in R \text{ and } a\neq 0\}$ and binary operation * define on G as: For $a,b,c,d \in R$, $(a,b)*(c,d)=(ac,bc+d)$. Show that if $H=\{(1,b)/b\in R\}$ then $H\leq G$. Where $(G,*)$ is group.

(2) $\sigma \epsilon\ S_6$ where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ then find σ^{-1} and $o(\sigma)$. Also Check whether σ is even or odd.

Q-5(A) Answer any two out of three. **[10]**

(1) State and prove Cayley's theorem.

(2) Prove that for finite cyclic group, order of group is same as order of it's generator.

(3) Prove that relation "isomorphism" between two groups is an equivalence relation.

Q-5(B) Answer any one out of two. **[04]**

(1) H and K be two normal subgroups of group G. if $H\cap K=\{e\}$ then for $\forall\ h\in H$ and $k\in K$ prove that $hk=kh$.

(2) Prepare group table in usual notation for quotient group S_3/A_3.

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